Math 229, Spring 2020
Project: Biological clocks

In this project you will study a non-linear system of DEs which is a standard model for a biological clock.

(Note: The following is heavily cribbed from the research statement of my friend and former colleague, Danny Forger, at the University of Michigan. The original model is due to Goodwin (1960).)

Three quantities are involved: the amounts of mRNA (X), an inactive protein (Y), and an activated protein (Z). The mRNA codes for and creates the inactive protein at a rate proportional to the amount of mRNA present. The inactive protein is activated at a rate proportional to the amount of inactive protein present. The active protein inhibits the production of the mRNA in some fashion – the rate of production of the mRNA is some function $f(Z)$. In addition to these rates of production, all three substances are cleared from the cell at rates proportional to the amount present (the constants of proportionality might be different).

Write a system of DEs encoding this model, and study its behavior as the different constants change, and as the function describing how Z inhibits the production of X changes.

In your writeup, please include the following:

1. The system of DEs that governs this model, and a description of how you came to it.

2. Based on the biological mechanism described above, give a qualitative account of how oscillations in this system might arise.

3. Assume that the effect of Z on X is $f(Z) = 1 - cZ$ for some positive constant c. Analyze the system qualitatively and find solutions in any way you can. Are there equilibria? Are there any oscillatory solutions? Under what conditions (i.e. what combinations of parameters) do these occur?

4. Based on the biological mechanism described above, make some predictions about how changing the effect of X on Z might change the

frequency of any oscillatory solutions. What if $f(Z)$ starts at 1 and decreases slowly (i.e. c is a small positive number)? Quickly (i.e. c is a large positive number)?

5. Test your predictions from the question above with some numerical solutions. You may also wish to try some non-linear functions for f and see how that affects things.