Math 229, Spring 2024
Project: Tsiolkovsky rocket equation

By Newton's second law, the force on an object is equal to its mass times its acceleration. In many situations, mass is constant, giving a simple first-order DE for the velocity of the object. But for a rocket, mass changes dramatically as it propels itself by exhausting combusted fuel. For this project, model this situation with a DE and find some solutions for the velocity of the rocket.

In your writeup, please include the following:

1. Write the general form of a first order DE for the velocity of the rocket in terms of time. Be sure to take account of which quantities could be a function of time.

2. Develop a simple model for the force provided by the rocket and the mass of the rocket as functions of time. Does your model make sense for an object that propels itself precisely by expelling fuel? Solve the corresponding differential equation. How does your solution compare to a solution with the same force and constant mass? Does this make sense?

3. Try some other fuel use strategies: use a lot at first, then taper off; start slow and build; etc. Find solutions. How do the solutions compare?

4. Can you formulate and prove a conjecture relating final velocity and amount of fuel?

5. Read Randall Monroe's (of xkcd fame) *What if?* post on rocket golf:

   http://what-if.xkcd.com/85/

   Derive from your simple model an equation for the total change in velocity in terms of the fuel use. This may be different from the Tsiolkovsky rocket equation Monroe uses. How does what you find compare to his work? What features of your model could be used to study the rocket golf question he is discussing?