

Math 229, Spring 2024
Project: Heating and cooling

Place an object at one temperature into an environment with constant external temperature T_{ext} . Newton's law of heating and cooling says that the rate of change in the temperature of the object as it heats up or cools down toward T_{ext} is proportional to the difference between its temperature and the external temperature.

Here are two situations in which this law applies:

- Suppose on a cold day the outside temperature is roughly constant. If you turn off the heat in your apartment, the law controls the rate at which your apartment cools.
- You put a cake at room temperature into the oven to bake. The law controls how fast the temperature of the rises, as well as how fast it cools when you remove the cake from the oven.

In this project you will use the heating/cooling DE to answer some practical questions about these two situations. First, what is the most energy-efficient way to use your AC if you will be out of the house for part of the day? Second, what is the best baking strategy for your cake?

In your writeup, please include the following:

1. Write down the DE governing the rate of change of temperature described by Newton's law. Explain the practical significance of any parameters that appear.
2. Solve the DE, and explain the qualitative effect for any parameters that appear on the solutions.
3. In the apartment example, let us assume that the rate of change in temperature is proportional to the rate at which heat energy leaves your apartment. If you run your heater all day to compensate for this loss of energy, how much energy is required to keep your apartment a constant temperature?
4. Alternately, suppose that you turn off your heater completely when you leave the apartment in the morning, and turn it back on when

you return home. Use the law to determine how much energy will be required to bring your apartment back to room temperature. Which strategy is better?

5. In the cake example, say that your cake batter should be raised to a temperature of at least 275°F for 15 minutes to cook completely. There are complications in cake baking, however. First, the parameter of proportionality in the heating equation is different for the center of the cake and for its edges (the center will warm more slowly). Second, when you remove the cake from the oven it continues to bake via residual heat until its temperature falls again. In addition, you do not want the cake to be in the oven for more than 45 min, else it will be too dry.

Given these constraints, use the heating and cooling law to devise a baking strategy that will fully cook the center of the cake, mitigate over-cooking the edges of the cake, and not dry it out as best you can. Your strategy will depend on the parameters for the edge and center of the cake, which should remain in your solution as constants.

(Note: This is a simplified model of how heat really moves through the cake, which would be better addressed by solving the *partial* differential equation known as the heat equation.)