Math 229, Spring 2024
Project: Smart Euler method

Euler's method has trouble with DEs like $y' = f(t, y)$ when f changes rapidly, such as

$$y' = e^t \sin(y).$$

Decreasing the step size improves accuracy, but increases run time for the algorithm. One way to deal with this is to use small step size when necessary and larger steps when the method is more accurate.

In this project, you will devise a 'smart' Euler method along these lines. Come up with some way to measure how inaccurate the Euler method is likely to be at a given point. Then incorporate this into a new algorithm which uses smaller step size at such points.

Some ability with computer programming so you can efficiently implement the new algorithm will be necessary for this project.

In your writeup, please include the following:

1. A description of how you measure the likely inaccuracy of Euler's method and a justification for your idea.

2. A full description of your algorithm.

3. A few tests of your algorithm on some DEs like the one above. Compare it to the usual Euler's method in terms of accuracy and runtime.

4. In addition, you should turn in your code or whatever other computing work you did for the project.